Calculation of Potential Flow Past Simple Bodies Using Axial Sources and a Least-Squares Method

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Nomenclature

a = semimajor axis of ellipsoid
A_{ij} = aerodynamic influence coefficient, Eq (5)
b = semiminor axis of ellipsoid
L = length of an arbitrary body
NC = number of control points on body surface
NS = number of sources along body axis
r_{ij} = distance from source to control point Eq (2)
R_{ij} = r_{ij}/a
S_j = nondimensional source strength, Eq (4)
u = horizontal velocity component at any point (x y)
U = freestream velocity
x = coordinate in direction of body axis
x_j = location of source on body axis
X_i = x_i/a
X_j = x_j/a
x_i y_i = location of control point on body surface
y = coordinate normal to body axis
Y_i = y_i/a
ϵ = ellipse ratio = b/a
Λ_j = strength of point source located at position (x_j, 0)
ψ = stream function

Introduction

ONE of the best known methods for calculating the potential flow about bodies of revolution is associated with von Karman's name [1] The method uses sources distributed along the axis of the body Recent computer oriented variations of this method have been used for the aerodynamics of external stores [2 4] Furthermore the addition of boundary layer and separation effects has been considered in recent papers [5 7]

The use of sources distributed along the axis of a body leads to a Fredholm integral equation of the first kind The most common approximation of this integral equation involves simultaneous equations for unknown source strengths Recent papers[3 8] have considered the number of control points (NC) different from the number of sources (NS) solving the resulting algebraic equations by the method of least squares The purpose of such an approach is to avoid or at least postpone some of the problems associated with ill conditioning of the equations approximating the Fredholm equation of the first kind

In the present Note results from a systematic variation in the number of sources (NS) and number of control points (NC) are summarized for ellipsoids of varying fineness ratio The stream function $\psi(x\ y)$ is evaluated at a large number of points on the surface of the body A small absolute value of ψ at all points identifies an accurate solution The accuracy criterion used by Jones[3] is a small value of the velocity normal to the body surface When either of these criteria are used one can decide whether the surface prssure distribution and the flow field will be correctly calculated for the par ticular source distribution along the axis of the body

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Calculation Method

The method used herein to calculate the flow about bodies is essentially the same as the method used in Ref 8 for planar shapes The only difference is in the Stokes stream function $\psi(x_i\ y_i)$ due to uniform flow and the sources located at points x_j on the body axis where $j = 1$ to NS

$$\psi = \frac{1}{2} U y_i^2 - \frac{1}{4\pi} \sum_{j=1}^{NS} \Lambda_j (x_i - x_j)/r_{ij} \tag{1}$$

$$r_{ij}^2 = (x_i - x_j)^2 + y_i^2 \tag{2}$$

Setting $\psi = 0$ on the body surface leads to linear algebraic equations for the nondimensional source strengths S_j

$$\sum_{j=1}^{NS} A_{ij} S_j = \frac{1}{2} Y_i^2 \tag{3}$$

in which

$$S_j = \Lambda_j / 4\pi U a^2 \tag{4}$$

$$A_{ij} = (X_i - X_j)/R_{ij} \tag{5}$$

$$R_{ij}^2 = (X_i - X_j)^2 + Y_i^2 \tag{6}$$

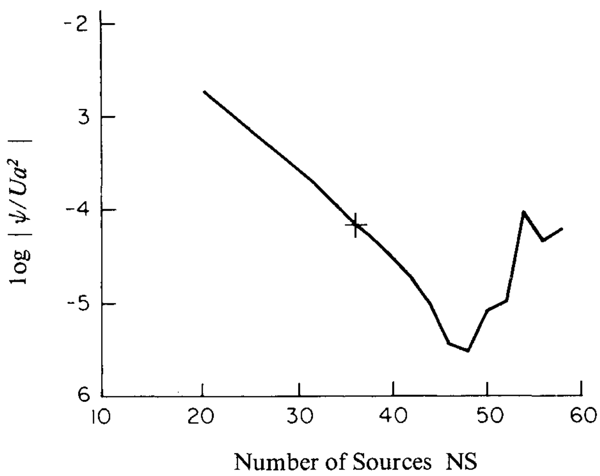

Fig 1 Maximum value of calculated stream function evaluated from ψ at 384 locations on ellipsoid; $\epsilon = 0\ 25$ and NC = 36 control points

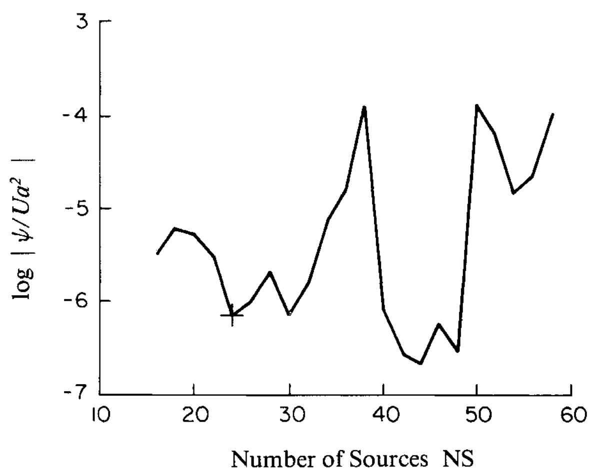

Fig 2 Maximum value of calculated stream function evaluated from ψ at 384 locations on ellipsoid; $\epsilon = 0\ 60$ and NC = 24 control points

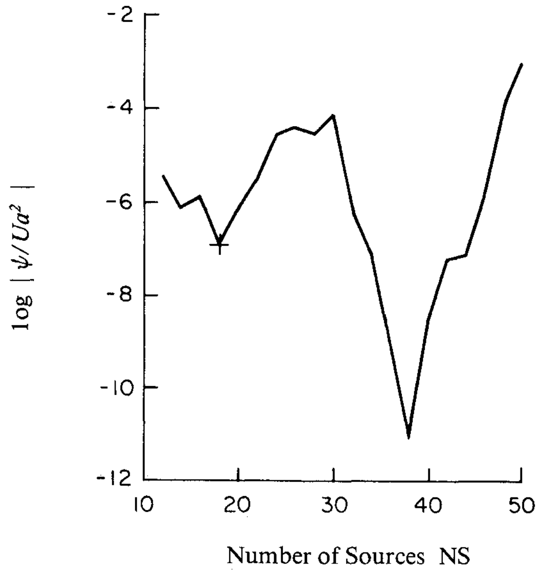

Fig 3 Maximum value of calculated stream function evaluated from ψ at 384 locations on ellipsoid; $\epsilon = 0$ 90 and NC = 18 control points

and all lengths are made nondimensional by the length a of the semimajor axis of the ellipsoidal body

Sources are distributed at equal intervals along the x axis The algebraic equations [Eq (3)] are satisfied at equally spaced control points on the body The number of control points NC is arbitrary so that the set of equations is in general, either underdetermined or overdetermined

Because the stagnation points are so important in the determination of flow around a body additional control points are located at the front and rear stagnation points At these points the horizontal velocity component u is set equal to zero where

$$u = \frac{1}{y}\frac{\partial \psi}{\partial y} \tag{7}$$

For the symmetric body shape used in these calculations NS/2 symmetry conditions are added giving

$$S_j + S_{\mathrm{NS}-j+1} = 0 \qquad j = 1\ 2 \qquad \mathrm{NS}/2 \tag{8}$$

All calculations were carried out using double precision (sixteen decimal digits) The solution of the overdetermined set of linear algebraic equations was obtained using IMSL Inc 's computer code routine LLSQF[9] for solving linear least squares problems

Results and Conclusions

This summary of results for ellipsoids at zero angle of attack is based on the following parameter variations: 1) ϵ the ellipse ratio b/a varies from 0 25 to 0 99 2) NC the number of control points varies from 12 to 50 and 3) NS the number of sources, varies from 12 to 58

Most of the general trends discussed in Ref 8 for a planar ellipse apply as well for the ellipsoid Thus for a fixed number of sources and control points numerical problems associated with ill conditioned equations are more critical for blunt bodies than for slender bodies as shown in Fig 2 of Ref 8 Furthermore the shape of the source distribution for blunt bodies is quite different from that for slender bodies as in Figs 8 and 9 of Ref 8

The primary new results of the present study are shown in Figs 1 to 3 These figures present stream function on the body vs number of sources NS for a given ellipse ratio ϵ and number of control points NC In calculating these results the stream function ψ on the body was evaluated at 384 points from nose to tail From these 384 values the one largest in magnitude is plotted for each value of NS

In evaluating the results in Figs 1 to 3 the point plotted for NS = NC serves as a useful reference point At this reference point the only difference between the present solution and one based on a set of simultaneous equations is the specification of stagnation points at $X = \pm 1$ and the set of symmetry conditions [Eq (8)] Over the range of parameters investigated the magnitude of ψ / Ua^2 is less than 10^{-4} for the reference cases If one adds more sources for a fixed number of control points there is a value of NS for which ψ is minimum In that way the solution of greatest accuracy is identified

It is clear from these figures that there is no reliable rule of thumb for deciding prior to calculation the number of sources which will result in greatest accuracy In view of this it would seem reasonable to first perform calculations for a particular body and assume NS = NC A check on the value of the stream function at many points on the body should then be performed If the resulting value $| \psi / UL^2 |$ is unacceptably large, then one might vary NS over the approximate range of NC 2NC until the minimum ψ solution is obtained.

It seems likely that the greatest potential gain from the use of the least squares approach would be achieved in com bination with other methods such as optimized source locations [3] polynomial distributions of source strength,[4 5 10 11] etc Obviously each additional feature complicates the solution method when compared with von Karman's original approach [1 2]

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